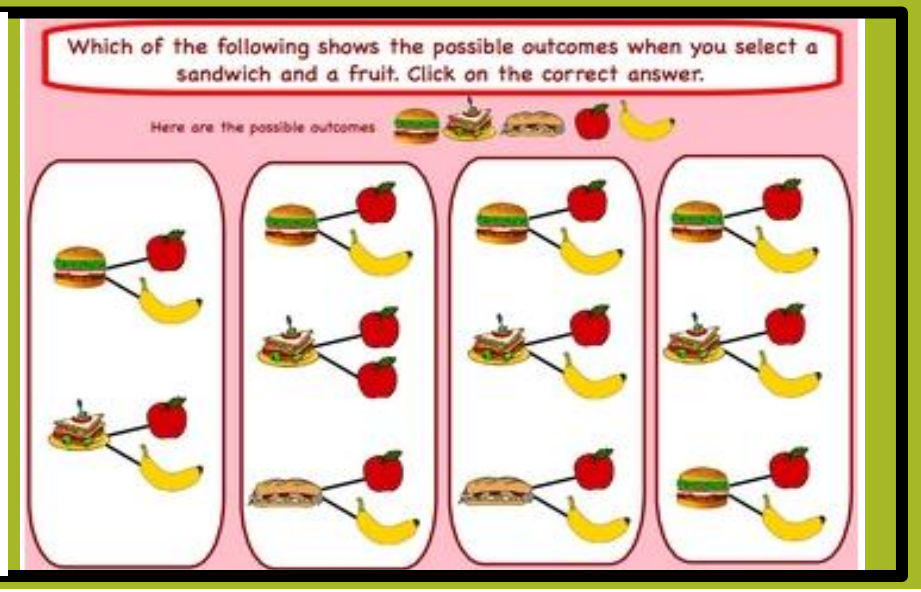
COMBINATIONS

MODULE-9

$$nCr = \frac{n!}{r!(n-r)!}$$



QUESTIONS……

At least ➞ minimum (≥)

Q1) In an examination, a question paper consists of 12 questions divided into two parts, Part I and Part II, containing 5 and 7 questions, respectively. A student is required to attempt 8 questions in all, selecting at least 3 from each part. In how many ways can a student select the questions?

Alternative	Part I (out of 5)	Part II (out of 7)	No. of ways
(i)	3	5	${}^5C_3 \times {}^7C_5$
(ii)	4	4	${}^5C_4 \times {}^7C_4$
(iii)	5	3	${}^5C_5 \times {}^7C_3$

Required no. of ways of making a selection

$= {}^5C_3 \times {}^7C_5 + {}^5C_4 \times {}^7C_4 + {}^5C_5 \times {}^7C_3$

$= \frac{5\times4}{1\times2} + \frac{7\times6}{1\times2} + 5 \times \frac{7\times6\times5}{1\times2\times3} + 1 \times \frac{7\times6\times5}{1\times2\times3}$

$= 10 \times 21 + 5 \times 35 + 1 \times 35$

$= 210 + 175 + 35 = 420$

Q2) Determine the no. of 5-card combinations out of a deck of 52 cards if each selection of 5 cards has exactly one king.

King ➞ 4
Remaining ➞ 48

No. of 5-card combinations = ${}^4C_1 \times {}^{48}C_4$

QUESTIONS……

At least ⟹ ≥ (min)
At most ⟹ ≤ (max)

Q3) A committee of 7 has to be formed from 9 boys and 4 girls. In how many ways can this be done when the committee consists of :

(i) exactly 3 girls ?

Boys	Girls
9	4

No. of ways of selecting exactly 3 girls

$= {}^{9}C_4 \times {}^{4}C_3 = \frac{9\times8\times7\times6}{4\times3\times2\times1} \times 4 = 504$

(ii) at least 3 girls?

Girls	Boys
3	4
4	3

No. of ways of selecting at least 3 girls

$= {}^{4}C_3 \times {}^{9}C_4 + {}^{4}C_4 \times {}^{9}C_3 = \frac{4!}{3!} \times \frac{9!}{4!5!} + 1 \times \frac{9!}{3!6!}$

= 504 + 84 = 588.

(iii) at most 3 girls?

Girls	Boys
0	7
1	6
2	5
3	4

No. of ways of selecting at most 3 girls

$= {}^{4}C_0 \times {}^{9}C_7 + {}^{4}C_1 \times {}^{9}C_6 + {}^{4}C_2 \times {}^{9}C_5 + {}^{4}C_3 \times {}^{9}C_4$

$= 1 \times \frac{9!}{2!7!} + 4 \times \frac{9!}{3!6!} + \frac{4!}{2!2!} \times \frac{9!}{5!4!} + 4 \times \frac{9!}{4!5!}$

= 36 + 336 + 756 + 504 = 1632.

QUESTIONS…..

Q4) We wish to select 6 persons from 8, but if the person A is chosen, then B must be chosen. In how many ways can the selection be made?

Case (i): **when person A is not chosen:**

6 persons can be selected from the remaining 7 persons in ${}^{7}C_{6}$ ways.

Case (ii): **when person A is chosen:**

Then person B has to be chosen. 4 more persons have to be chosen from the remaining 6 in ${}^{6}C_{4}$ ways.

Total no. of selection = ${}^{7}C_{6} + {}^{6}C_{4}$

$$= 7 + \frac{6!}{2!4!} = 7 + 15 = 22$$

Q5) From a class of 25 students, 10 are to be chosen for an excursion party. There are 3 students who decide that either all of them join or none of them will join. In how many ways can the excursion party be chosen?

Case (i): **If all 3 students are included:**

7 more students have to be chosen from the remaining 22 students in ${}^{22}C_{7}$ ways.

Case (ii): **If all 3 students are not included:**

10 students have to be chosen from the remaining 22 students in ${}^{22}C_{10}$ ways.

∴ Required no. of ways = ${}^{22}C_{7} + {}^{22}C_{10}$

QUESTIONS…….

Q6) In how many ways can a committee of 5 persons with a chairperson be selected from 12 persons?

No. of ways of selecting a chairperson from

12 persons = ${}^{12}C_1$ ways.

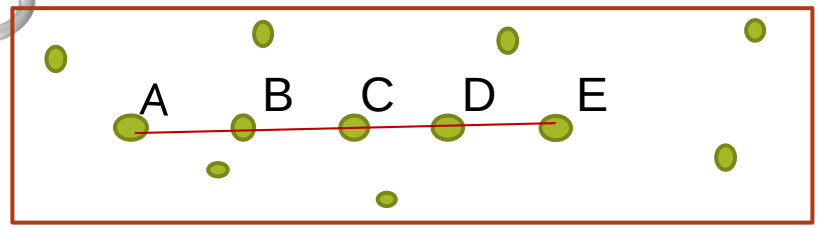
And the remaining 4 persons can be selected from the remaining 11 in ${}^{11}C_4$ ways.

∴ Total no. of ways = ${}^{12}C_1$ x ${}^{11}C_4$ And ➡ ×

$$= 12 \times \frac{11\times10\times9\times8}{4\times3\times2\times1}$$

= 12 x 330 = 3960.

Q7) Out of 12 points in a plane, no three are in the same line except five points which are collinear. Find the no. of lines that can be formed.



No. of lines formed from 12 points taking 2 at a time ${}^{12}C_2$

No. of lines formed from 5 points taking 2 at a time = ${}^{5}C_2$

But 5 collinear points, when joined pairwise,

results in only one line.

Required no. of straight lines = ${}^{12}C_2$ - ${}^{5}C_2$ +1

QUESTIONS…..

Q 8). If ${}^{n}C_{r-1} = 36$ ${}^{n}C_{r} = 84$ and ${}^{n}C_{r+1} = 126$, then find the value of ${}^{r}C_{2}$.

Sol. We know that $\frac{{}^{n}C_{r}}{{}^{n}C_{r-1}} = \frac{n-r+1}{r}$

$$\therefore \quad \frac{n-r+1}{r} = \frac{84}{36} \text{ (given)}$$

$$\Rightarrow \quad \frac{n-r+1}{r} = \frac{7}{3} \quad \Rightarrow 3n - 3r + 3 = 7r$$

$$\Rightarrow \quad 10r - 3n = 3 \quad \text{-------(i)}$$

$$\frac{{}^{n}C_{r+1}}{{}^{n}C_{r}} = \frac{n-(r+1)+1}{r+1} = \frac{126}{84} \text{ (given)}$$

$$\therefore \quad \frac{n-r}{r+1} = \frac{3}{2} \quad \Rightarrow 2n - 2r = 3r + 3$$

$$\Rightarrow \quad 2n - 5r = 3 \quad \text{---------(ii)}$$

Solving (i) and (ii), we get $n = 9$ and $r = 3$.

$$\therefore \quad {}^{r}C_{2} = {}^{3}C_{2} = 3$$

Q9) Find the no. of ways in which we can choose a committee from four men and six women, so that the committee includes at least two men and exactly twice as many women as men.

Men	Women
2	4
3	6

Required no. of committee formed

$$= {}^{4}C_{2} \times {}^{6}C_{4} + {}^{4}C_{3} \times {}^{6}C_{6}$$

$$= \frac{4\times3}{2} \times \frac{6\times5}{2} + 4 \times 1$$

$$= 6 \times 15 + 4 = 94$$

QUESTIONS…..

Q 10) In how many ways can we get exactly 4 hearts or exactly 3 spades in a draw of 6 cards?

Exactly 4 hearts → $^{13}C_4$

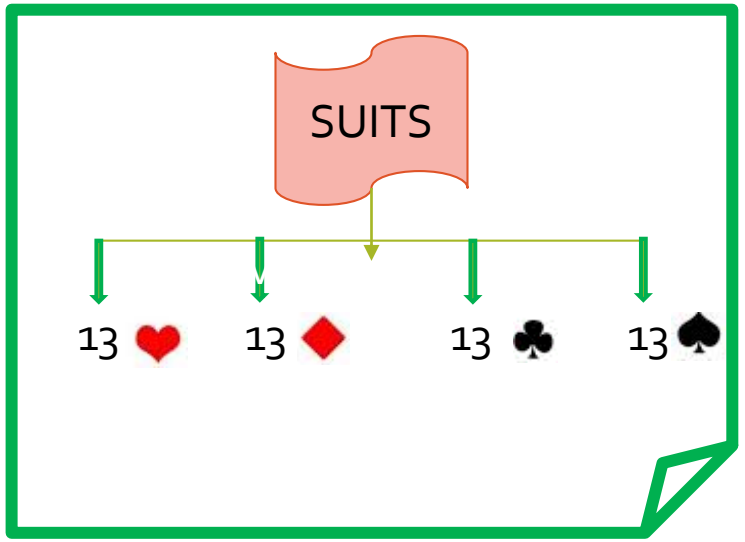
Remaining 2 cards → $^{39}C_2$

AND → x

OR → +

Exactly 3 spades → $^{13}C_3$

Remaining 3 cards → $^{39}C_3$



∴ No. of ways of drawing exactly 4 hearts or 3 hearts

$= {}^{13}C_4 \times {}^{39}C_2 + {}^{13}C_3 \times {}^{39}C_3$

HOMEWORK QUESTIONS……

1) There are 7 boys and 4 girls in a class. In how many ways can a team of 5 members be selected if the team has (i) no girls (ii) at least one boy and one girl (iii) at least 3 girls?

2) A committee of 3 members is to be formed out of 5 men and 2 women. Find the no. of ways of selecting the committee, if it is to consist of at least one woman.

3) In an examination, a question paper consisting of 10 questions is divided into two parts A and B, each part consisting of 5 questions. A student is required to attempt 6 questions in all, taking at least 2 questions from each part. In how many ways can the student select the questions?

4) Three balls are drawn from a bag containing 5 red, 4 white and 3 black balls. Find the no. of ways in which this can be done if at least two are red balls.

5) If ${}^{n}P_{r} = 840$ and ${}^{n}C_{r} = 35$ find 'n' and 'r'.

ANSWERS:

Q1) (i) 21 (ii) 441 (iii) 91 Q2) 25 Q3) 200 Q4) 80 Q5) r = 4, n = 7.